Section #10.4 – Exponential Growth and Decay

What is the solution to the differential equation? What then about the equation with \(k\) a constant.

\[
\frac{dy}{dx} = y \quad \frac{dy}{dx} = ky
\]

Find the general solutions to each of the following differential equations. For differential equation (A), find the particular solution satisfying \(y = 50\) when \(t = 0\).

(A) \(\frac{dy}{dt} = 0.05y\)  
(B) \(\frac{dP}{dt} = -0.3P\)  
(C) \(\frac{dw}{dz} = 2z\)  
(D) \(\frac{dw}{dz} = 2w\)

Valproic acid is a drug used to control epilepsy; its half-life in the human body is about 15 hours.

(A) Use the half-life to find the constant \(k\) in the differential equation \(\frac{dQ}{dt} = -kQ\), where \(Q\) represents the quantity of drug in the body \(t\) hours after the drug is administered.

(B) At what time will 10% of the original dose remain?

The amount of ozone, \(Q\), in the atmosphere is decreasing at a rate proportional to the amount of ozone present. If time \(t\) is measured in years, the constant of proportionality is -0.0025. Write a differential equation for \(Q\) as a function of \(t\), and give the general solution for the differential equation. If this rate continues, approximately what percent of the ozone in the atmosphere now will decay in the next 20 years?

The amount of land in use for growing crops increases as the world’s population increases. Suppose \(A(t)\) represents the total number of hectares of land in use in year \(t\): (A hectare is about 2 acres.)

(A) Explain why it is plausible that \(A(t)\) satisfies the equation \(A'(t) = kA(t)\): What assumptions are you making about the world’s population and its relation to the amount of land used?

(B) In 1950 about \(1 \times 10^9\) hectares of land were in use; in 1980 the figure was \(2 \times 10^9\). If the total amount of land available for growing crops is thought to be \(3.2 \times 10^9\) hectares, when does this model predict it is exhausted? (Let \(t = 0\) in 1950.)