Math 131 Exam II "Sample Questions"

This is a compilation of exam II questions from old exams (written by various instructors). They cover chapters 2 and 3. The solutions can be found at the end of the document.

1. The graph of the derivative of \( f(x) \) is shown. Which choice best represents the graph of the original function, \( f(x) \)? Which represents the graph of \( f''(x) \)?

![Graph of the derivative, \( f'(x) \)](image1.png)

a) b) c) d) e)

2. The table below gives the population in hundreds of thousands of a city over a period of years.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>3.0</td>
<td>2.8</td>
<td>2.7</td>
<td>2.6</td>
<td>2.7</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

a) Find the average rate of change of the population with respect to time (in years) from 1987 to 1990. Interpret your answer.

b) Estimate the instantaneous rate of change for 1988. Interpret your answer.

3. An object is traveling with a velocity given by \( f(t) = 2t + 1 \) on \([0, 2]\). Find an estimate for the distance traveled using a right hand sum with \( n = 4 \) rectangles. Is this a lower or upper estimate?

4. If \( f(x) = 4x - x^2 \) and \( g(x) = 2x \), then what is the area of the region enclosed between the two graphs?

5. Find the area between the function \( f(x) = (x - 2)^3 \) and the \( x \)-axis from \( x = 1 \) to \( x = 4 \). Draw the graph and shade the appropriate region.
6. If you know that \( f(x) \) is continuous on \([5, 6]\) and \( F'(x) = f(x) \) where \( F(5) = 9, F(6) = 12, f(5) = 6, \) and \( f(6) = 16, \) then what would the value of the following integral be?

\[
\int_{5}^{6} f(x)dx
\]

7. Given \( f(x) = \frac{6}{x}, \) find \( f'(x) \) using the limit definition of derivative.

8. An object travels with a velocity (in ft/sec) given by the following table. Give an upper and lower estimate for the distance traveled by the object. Explain what you are doing.

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>17</td>
</tr>
</tbody>
</table>

9. Given the position function \( s(t) = 28.7 + 36.4 \ln(t) \), where \( t \) is the time in seconds and \( s(t) \) is in feet,

a) estimate the velocity at \( t = 3 \) seconds.

b) Is the velocity greater at \( t = 3 \) seconds or \( t = 4 \) seconds?

c) Is the acceleration positive or negative at \( t = 3 \) seconds? What does this mean?

10. The graph of \( y = f(x) \) is given below.

![Graph](image)

a) Find \( \int_{-4}^{0} f(x)dx \).

b) If the area of the shaded region is equal to \( 2A \), what is \( \int_{-4}^{4} f(x)dx \)?
11. Given is the graph of \( f(x) \). Which of the following statements is (are) true?

I. \( f(2) > f'(3) \)
II. \( f'(3) < f'(4) \)
III. \( f''(4) > f(3) \)

12. Find the average rate of change of \( f(x) = 2x^3 + 4x \) on \([-1, 1]\).
   a) 12  
b) 16  
c) 0  
d) 6  
e) None of the above

13. If \( f(2) = 3 \) and \( f'(2) = -1 \), find the equation of the tangent line to \( f \) when \( x = 2 \).

14. The figure given in the graph is the second derivative of a function, \( f(x) \). Choose the correct graph of \( f(x) \).

   graph of \( f''(x) \)
   a) 
   b) 
   c) 
   d) 
   e) none of the above

15. Find the area of the region bounded by \( y = (x - 1)^2 + 1 \) and the x-axis between \( x = -1 \) and \( x = 2 \).
16. The graph of \( y = f(x) \) is given below. Sketch the graphs of \( f'(x) \) and \( f''(x) \).

17. Suppose you buy some hot chocolate at a cold Aggie Football game. When you buy it, its temperature is 170°F. The temperature outside is 50°F and the rate at which the temperature is changing is given by \( r(t) = -5e^{-0.1t} \), with \( t \) in minutes and \( r(t) \) in degrees per minute. Estimate the temperature of your hot chocolate after 10 minutes.

   a) 128°F  
   b) 31.6°F  
   c) 138.4°F  
   d) 88.4°F  
   e) None of the above

18. Given the graph of \( f(x) \), which of the following is the largest?

   a) \( f''(2) \)  
   b) \( f(0) \)  
   c) \( f'(0) \)  
   d) \( f'(-2) \)  
   e) \( f(-1) \)

19. If \( t \) is measured in minutes and \( R(t) \) is in feet per minute, then what are the units for \( \int_0^{60} R(t) \, dt \)?

20. Find the area between the two curves \( y = x^2 - 4 \) and \( y = 2x + 4 \). Draw the graphs of the curves and shade the appropriate area.
21. Given the function \( f(x) = x^3 - 8x^2 + 12x \), answer parts a) - c):
   a) What are the roots of this function?
   
   b) Evaluate \( \int_{0}^{6} f(x)dx \) and explain why your answer has the sign (positive or negative) that it does.
   
   c) Find the area bounded by the x-axis and \( f(x) = x^3 - 8x^2 + 12x \).

22. If \( F'(x) = -9x^2 - 12x + 2 \) and \( F(0) = 0 \), answer the following:
   a) Approximate \( F(2) \) using the Fundamental Theorem of Calculus.
   
   b) Use the graph of \( F'(x) \) to determine the intervals of increase and decrease for the function \( F(x) \).

23. Use the definition of derivative to find the derivative of \( f(x) = x^2 - 5x \).

24. The data below gives the rate of change in the number of farmers moving into a particular community per year:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>farmers per year</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>17</td>
<td>46</td>
</tr>
</tbody>
</table>

   a) If \( x \) is the number of years since 1972 and \( f(x) \) is the rate of change in the number of farmers, use the data to find upper and lower estimates of \( \int_{0}^{4} f(x)dx \).
   
   b) A model for the data is \( f(x) = x^3 - x^2 - x + 2 \). Use this to approximate \( \int_{0}^{4} f(x)dx \) using Riemann sums and \( n = 8 \) rectangles.
   
   c) It can be shown that the derivative of \( F(x) = \frac{1}{4}x^4 - \frac{1}{2}x^3 - \frac{1}{2}x^2 + 2x \) is our model, \( f(x) \). Find the exact value of \( \int_{0}^{4} f(x)dx \).
1. \( f(x) \) is a) and \( f''(x) \) is d)
2. a) 6700 people per year. From 1987 to 1990, the population was increasing at an average of 6700 people per year.
b) 0 people per year. In 1988 the population was not decreasing nor increasing. It is at a minimum of 2,600,000 people.
3. 7, upper
4. 4/3
5. 4.25
6. 3
7. \(-\frac{6}{x^2}\)
8. lhs = 25 feet and rhs = 42 feet
9. a) 12.13 feet per second
   b) at \( t = 3 \)
   c) negative; the velocity is decreasing at \( t = 3 \) (actually decreasing for \( t > 0 \))
10. a) -3
    b) -1 - A
11. I only
12. D
13. \( y = -x + 5 \)
14. A
15. 6
16. graph of \( f'(x) \) and the graph of \( f''(x) \):
17. C
18. C
19. feet
20. area = 36
21.  
a) $x = 0, 2, 6$
b) -36. More of the graph lies below the $x$-axis than above it.
c) 49.33

22.  
a) -44
b) $F$ is decreasing on $(-\infty, -1.48) \cup (0.15, \infty)$
and $F$ is increasing on $(-1.48, 0.15)$.

23.  
$2x - 5$

24.  
a) lhs = 24 farmers, rhs = 68 farmers
b) lhs = 32.5 or 33 farmers, rhs = 54.5 or 55 farmers
c) $42 \frac{2}{3}$ or 43 farmers