Week In Review #10 - Test 3 Review
Covers: Chapter 5 and Chapter 7

♦ This review gives one or two examples from each section. It is NOT a thorough review by itself, but rather some additional practice problems that you can study along with your homework and the lecture notes.

♦ The problems in Week-In-Review 8, and 9 are also good review problems for the exam.

Review Problems:
1. The following table gives the emissions, $E$, of nitrogen oxides in millions of metric tons per year in the US. Let $t$ be the number of years since 1940 and $E = f(t)$.

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<tbody>
<tr>
<td>$E$</td>
<td>6.9</td>
<td>9.4</td>
<td>13.0</td>
<td>18.5</td>
<td>20.9</td>
<td>19.6</td>
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(a) What are the units and meaning of $\int_{0}^{50} f(t) dt$?

(b) Estimate $\int_{0}^{50} f(t) dt$.

2. Your velocity is $v(t) = \ln(t^2 + 1)$, meters per minutes, for $0 \leq t \leq 2$ hours. Find the distance traveled during this time.
3. A bat starts out traveling towards the exit of a tunnel 90 feet away. The graph below describes the bats velocity vs. time.
(a) When does the bat change the direction?
(b) How far from the starting point is the bat after 6 seconds?
(c) How far has the bat traveled in the 6 seconds?
(d) Does the bat make it out of the tunnel in 10 seconds?

4. (a) Estimate $\int_{1}^{4} \sqrt{x} \, dx$ using a left-hand-sum with $\Delta x = 0.5$.
(b) Represent the answer in (a) graphically.
(c) Use a calculator to find the value of $\int_{1}^{4} \sqrt{x} \, dx$

5. For the following graph, set up the integral(s) that you would use to find the shaded area.
6. For the function \( g(x) = \frac{3^x}{4} - 3 \).

(a) Evaluate the definite integral for the given function from \( x = -1 \) to \( x = 5 \).
(b) Find the area between the function and the \( x \)-axis from \( x = -1 \) to \( x = 5 \).

7. Determine the value of \( b \) if the area under the graph of \( f(x) = \frac{2}{x} \) between \( x = 1 \) to \( x = b \) is 4. Assume \( b > 1 \).

8. Consider the following graph of \( f(x) \), arrange the following integrals in increasing order.

\[
\begin{align*}
A &= \int_{-1}^{2} f(x) \, dx \\
B &= \int_{-1}^{1} f(x) \, dx \\
C &= \int_{0}^{3} f(x) \, dx \\
D &= \int_{-3}^{0} f(x) \, dx
\end{align*}
\]
9. The following graph is the graph of \( g'(x) \), with some areas labeled.

(a) Find \((x, y)\) coordinates for all local maxima and minima.
(b) Estimate the coordinates of the inflection points if they exist.
(c) If \( g(0) = 1.5 \), sketch the graph of \( g(x) \). Be sure to label.

10. The net worth, \( f(t) \), of a company is growing at a rate of \( f'(t) = 2000 - 12t^2 \) dollars per year, where \( t \) is in years since 1990.

(a) Find the total change of net worth of the company between 1990 and 2000 exactly.
(b) If the company is worth $40,000 in 1990, what is it worth in 2000?
11. If $F'(x) = 5 \sin(3x^2)$ and $F(-1) = -1$, find $F(1)$.

Find the following indefinite integrals:

12. $\int \left(x^2 + \frac{1}{3x^3} - e\right) dx$

13. $\int \left(4 + e^{-4x} - \cos(4x)\right) dx$

14. $\int \frac{\sqrt{y} + 2y}{y^2} dy$
15. \[ \int (x^3 + 2)(x^4 + 8x + 3)^{1/3} \, dx \]

16. \[ \int \left[ \frac{t + 1}{t^2 + 2t - b} - \cos(5t) \right] \, dt \]

17. \[ \int \frac{e^{y^2}}{x^2} \, dx \]
18. \[ \int \frac{e^x - e^{-x}}{(e^x + e^{-x})^3} \, dx \]

19. \[ \int \sin^6(5\theta) \cos(5\theta) \, d\theta \]

20. Evaluate the definite integral EXACTLY: \[ \int_0^1 t^2 e^{-t^3} \, dt \]
Answers:
1. (a) The integral \( \int_{0}^{50} f(t)dt \) represents the total emissions of nitrogen oxides, in millions of metric tons, during the period 1994 to 1990.
   (b) 750.5 million metric tons
2. 912.1 meters
3. (a) \( t = 5, 7, 10 \) seconds  \( \) (b) 70 ft  \( \) (c) 90 ft  \( \) (d) Yes.
4. (a) LHS = 4.4115  \( \) (b) 4.6667
5. The area \( = \int_{0}^{2} [(g(x) - h(x)]dx + \int_{2}^{5} [(g(x) - f(x)]dx \)
6. (a) 37.22  \( \) (b) 51.48
7. \( b = e^2 \)
8. \( D < B < A < C \)
9. (a) Local maximum: \((1.3, 5)\)
   Local minimum: \((4.6, -3)\)
   (b) Inflection point: \((3, 1)\)
   (c) Sketch the graph.
10. (a) $16,000  \( \) (b) $56,000
11. 4.15
12. \( \frac{1}{3} x^3 - \frac{1}{6} x^2 - ex + C \)
13. \( 4x - \frac{1}{4} e^{-4x} - \frac{1}{4} \sin(4x) + C \)
14. \( -\frac{2}{\sqrt{y}} + 2\ln|y| + C \)
15. \( \frac{3}{16} (x^4 + 8x + 3)^{4/3} + C \)
16. \( \frac{1}{2} \ln |t^2 + 2t - b| - \frac{1}{5} \sin(5t) + C \)
17. \( -e^{\sqrt{t}} + C \)
18. \( -\frac{1}{2(e^x + e^{-x})^2} + C \)
19. \( \frac{1}{35} (\sin(5\theta))^7 + C \)
20. \( \frac{1}{3} (1 - \frac{1}{e}) \)

If you find any mistakes, please let me know. Thanks!  \( li-chen2@neo.tamu.edu \)