1. [7 pts.] Find the area between $y = 2 \cos(2x)$ and $y = 2x^2$. (Set up and do integral by hand–you may use your calculator for the simple calculations in the end.)

Graphing the 2 functions, we see that the 2 points of intersection occur approximately when $x = -0.601$ and when $x = 0.601$. We also must subtract the bottom curve ($2x^2$ in this case) from the top curve ($2 \cos 2x$ in this problem). Thus, we have

$$\int_{-0.601}^{0.601} (2 \cos(2x) - 2x^2) \,dx \approx 1.576$$

2. [4 pts.] If $t$ is in months and $r(t)$ is in dollars per month, what are the units of $\int_{0}^{24} r(t) \,dt$? Also, draw a possible sketch for the integral. (Make sure EVERYTHING is labeled–axes, function, area, ...)

Here, the $x$-axis is in months, and the $y$-axis is in dollars/month. Multiplying these two together, we have $\text{dollars/month} \cdot \text{month} = \text{dollars}$

3. The total cost in dollars to produce $q$ units of a product is $C(q)$. Fixed costs are $20,000. The marginal cost is

$$C'(q) = 0.005q^2 - q + 56.$$  

(a) [3 pts.] Find the total cost to produce 174 units. (set up by hand–may use calculator to evaluate)

Total cost = Fixed costs + Variable costs
= $20,000 + \int_{0}^{174} (0.005q^2 - q + 56) \,dq \approx 23,386.04$

(b) [3 pts.] Find $C(175)$ using $C'(q)$. Interpret your answer. (set up by hand–may use calculator to evaluate)

$$C'(174) \approx 33.38$$

This means that it costs approximately $33.38 to make the 175th item. So the total cost, $C(175)$ is approximately $23,386.04 + 33.38 = 23,419.42$. (You could also have taken the integral of $C'(q)$ from 0 to 175.)
4. The figure below shows the derivative $G'(x)$. If $G(3) = 7$:

(a) [3 pts.] Find $G(0)$, $G(1)$, and $G(4)$. (Show how you get each answer!)

$G(0) - 6.3 + 8.2 = 7 \implies G(0) = 5.1$
$G(1) + 8.2 = 7 \implies G(1) = -1.2$
$G(3) - 1.7 = G(4) \implies 7 - 1.7 = G(4) = 5.3$

(b) [4 pts.] Label all local maxima and minima, global maxima and minima and inflection points of $G(x)$ on the graph of $G'(x)$.

5. [6 pts.] Find an antiderivative $F(x)$ if $F\left(\frac{\pi}{4}\right) = -1$ and $F'(x) = f(x)$. (set up and work completely by hand–may use calculator at very end for simple calculations)

$f(x) = \sin^3(x)\cos(x)dx$

First we need to find $\int (\sin x)^3 \cos x dx$. Let $u = \sin x$. Then $du = \cos x dx$. So now we have

$$\int u^3 du = \frac{u^4}{4} + C = F(x) = \frac{1}{4}(\sin x)^4 + C$$

Now we need to evaluate $F(x)$ at $x = \frac{\pi}{4}$.

$$F\left(\frac{\pi}{4}\right) = \frac{1}{4}\left(\sin\frac{\pi}{4}\right)^4 + C = -1$$
$$\implies \frac{1}{4}\left(\frac{\sqrt{2}}{2}\right)^4 + C = -1$$
$$\implies \frac{1}{4}\cdot\frac{16}{16} = -1$$
$$\implies \frac{1}{16} + C = -\frac{16}{16}$$
$$\implies C = -\frac{17}{16}$$

Thus,

$$F(x) = \frac{1}{4}\sin^4 x - \frac{17}{16}$$
6. [3 pts.] Using the figure below, if $F(x)$ is an antiderivative of $f(x)$ and $F(0) = 25$, estimate $F(7)$ using the FTC.

\[
\int_0^7 f(x) \, dx = F(7) - F(0)
\]

\[
F(7) = \int_0^7 f(x) \, dx + 25
\]

\[
\approx 2(-11.5) + 2(2) + 25
\]

\[
= 6
\]

7. [7 pts.] Find the exact area bounded by $y = 2x^3 + 5x^2 - 1$ and the $x$-axis. Graph and shade this area. (Set up integral(s) by hand–everything else can be done on the calculator.)

\[
\text{Area} = \int_{-5}^{-2.414} (2x^3 + 5x^2 - 1) \, dx + \int_{-5}^{4.14} (2x^3 + 5x^2 - 1) \, dx
\]

\[
\approx 4.3752 + 0.603965
\]

\[
= 4.979
\]

8. (a) [4 pts.] Estimate $\int_0^3 f(t) \, dt$ using Riemann sums and the following table. (set up by hand–may use calculator for calculations)

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<tbody>
<tr>
<td>$E$</td>
<td>6.9</td>
<td>9.4</td>
<td>13.0</td>
<td>18.5</td>
<td>20.9</td>
<td>19.6</td>
</tr>
</tbody>
</table>
l-h-s: 10(6.9+9.4+13) = 293
r-h-s: 10(9.4+13+18.5) = 409
Averaging the two together, we have \(\frac{293+409}{2} = 351\)

(b) [1 pt.] What is \(n\)? 3

(c) [1 pt.] What is \(\Delta r\)? 10

(d) [4 pts.] Represent the right-hand-sum of the above integral graphically.

9. Integrate the following problems. Simplify completely with coefficients in front and positive exponents!

(a) [7 pts.]

\[
\int \frac{\sqrt{2 + \sqrt{x}}}{\sqrt{x^2}} \, dx
\]

Let \(u = 2 + x^{1/3}\). Then \(du = \frac{1}{3}x^{-2/3} \, dx \Rightarrow 3du = x^{-2/3} \, dx\). So we have

\[
\int u^{1/2} \cdot 3 \, du = 3 \int u^{1/2} \, du = \frac{3u^{3/2}}{2} + C = 2u^{3/2} + C = 2(2 + \sqrt[3]{3})^{3/2} + C
\]

(b) [7 pts.]

\[
\int \frac{5}{(2e^{-x} + 1)e^x} \, dx
\]

Let \(u = -2e^{-x} + 1\). Then \(du = 2e^{-x} \, dx \Rightarrow \frac{1}{2}du = e^{-x} \, dx\). So we have

\[
5 \int \frac{1}{u} \cdot \frac{1}{2} \, du = \frac{5}{2} \int \frac{1}{u} \, du = \frac{5}{2} \ln|u| + C = \frac{5}{2} \ln \left| \frac{2}{e^x + 1} \right| + C
\]
(c) [7 pts.]

\[ \int \frac{5}{x} \, dx \]

\[ \int \frac{5}{x} \, dx = 5 \int \frac{1}{x} \, dx = 5 \ln|x| + C \]

(d) [7 pts.]

\[ \int \left( \frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x^3}} \right) \, dx = \int \left( \frac{1}{3} x^{-2/3} - \frac{1}{2} x^{-3/2} \right) \, dx = \frac{1}{3} x^{1/3} - \frac{1}{2} \cdot \frac{x^{-1/2}}{-1/2} + C = x^{1/3} + \frac{1}{\sqrt{x}} + C \]

10. Consider the improper integral \( \int_0^\infty e^{-x} \, dx \).

(a) [4 pts.] Find \( \int_0^b e^{-x} \, dx \) using the FTC.

\[ \int_0^b e^{-x} \, dx = \left. \frac{e^{-x}}{-1} \right|_0^b = -e^{-b} + e^0 = -e^{-b} + 1 \]

(b) [2 pts.] Evaluate (a) when \( b = 100, 1000, 10,000 \). What does this tell you about the improper integral?

(set up by hand–may use calculator to evaluate)

\[-e^{-100} = 1; \quad -e^{-1000} = 1; \quad -e^{-10,000} = 1\]

Thus

\[ \int_0^\infty e^{-x} \, dx \to 1. \]

11. [4 pts.] List the following integrals in descending order based on the graph below. Also tell whether each integral is positive, negative, or approximately 0.

\[ \int_c^e f(x) \, dx \text{ (pos)} > \int_b^d f(x) \, dx \text{ (pos)} > \int_a^c f(x) \, dx \text{ (pos)} > \int_a^c f(x) \, dx \text{ (neg)} \]
A bicyclist is pedaling along a straight road for one hour with a velocity $v$ shown in the figure below. She starts out eight kilometers from the lake and positive velocities take her towards the lake.

(a) [3 pts.] Does the cyclist ever turn around? If so, at what time(s)?

Yes, after 20 minutes.

(b) [3 pts.] When is she going the fastest? How fast is she going then? Toward the lake or away?

After 40 minutes, she is going away from the lake at 25 km/hr.

(c) [3 pts.] When is she closest to the lake? Approximately how close to the lake does she get?

After 20 minutes, she is approximately $8 - \frac{5}{3} = 6\frac{1}{3}$ km from the lake. (Each rectangle is $\frac{1}{6} \times 5 = \frac{5}{6}$.)

(d) [3 pts.] When is she farthest from the lake? Approximately how far from the lake is she then?

She is furthest from the lake after 1 hour. She is approximately $8 - \frac{5}{3} + 13\left(\frac{1}{8}\right) \approx 17.17$ km away.