1. A school library opened in 1980. In January of 2000, they had 10,000 books. One year later, they had 10,780 books. Assuming they acquire the same number of books at the start of each month, give a linear formula for the number of books, \( N \), in the library as a function of the number of years, \( t \), the library has been open.

2. A. Which two lines in the following figure have the same slope? Enter your answer as "I and II," etc.

   B. Which two lines have the same \( y \)-intercept?

   C. Which line has the largest slope?

   D. Which line has the largest \( y \)-intercept?

![Graph of four lines](image)

3. Find a formula for the linear function \( f \).

   \[
   \begin{array}{cccc}
   x & 0 & 5 & 10 & 15 \\
   f(x) & 10 & 25 & ? & ? \\
   \end{array}
   \]

   A) \( f(x) = 3x + 10 \)
   B) \( f(x) = 5x + 10 \)
   C) \( f(x) = 25x + 10 \)
   D) \( f(x) = 5x + 10 \)

4. A population is growing according to the formula \( P = 225(1.06)^t \), where \( P \) is the population at year \( t \). What is the initial population?

   A) 331
   B) 239
   C) 106
   D) 225

5. A population is growing according to the formula \( P = 325(1.06)^t \), where \( P \) is the population at year \( t \). What is the annual growth rate?

   A) 3.25%
   B) 3.45%
   C) 6%
   D) 12.36%
6. A population is growing according to the formula \( P = 200(1.06)^t \). What is the population in year 11?
   A) 380
   B) 2,332
   C) 3,888
   D) 202

7. A town has 1000 people initially. Find the formula for the population of the town, \( P \), in terms of the number of years, \( t \), if the town grows by 50 people a year.
   A) \( P = 1000 + 50t \)
   B) \( P = 1000(50)^t \)
   C) \( P = 1000(0.5)^t \)
   D) \( P = 1000 + (0.5)^t \)

8. A town has 1000 people initially. Find the formula for the population of the town, \( P \), in terms of the number of years, \( t \), if the town grows at an annual rate of 10% a year.
   A) \( P = 1000 + 10t \)
   B) \( P = 1000(0.1)^t \)
   C) \( P = 1000(1.1)^t \)
   D) \( P = 1000 + (1.1)^t \)

9. The following tables show values for two functions. Which one(s) could be exponential?

   \[
   \begin{array}{c|c|c|c}
   t & -2 & -1 & 0 \\
   f(t) & 250 & 300 & 360 \\
   \hline
   t & 2 & 3 & 4 \\
   g(t) & 500 & 750 & 1500 \\
   \end{array}
   \]

   A) \( f(t) \)
   B) \( g(t) \)

10. A substance has a half-life of 43 years. What percent of the original amount of the substance will remain after 20 years? Round to the nearest percent.

11. Solve \( 7^x = 11 \) for \( x \) using logs. Round to 3 decimal places.

12. Each of the curves in the following figure represents the balance in a bank account at time \( t \) after a single deposit at time \( t=0 \). Assuming continuously compounded interest, which curve represents the smallest initial deposit?

13. A standard cup of coffee contains about 100 mg of caffeine, and caffeine leaves the body at a rate of about 17% an hour. How many mg of caffeine are left in the body after 6 hours if this rate is continuous? Round to 2 decimal places.
14. What is the equation for the graph obtained by shifting the graph of \( y = x^3 \) vertically upward by 6 units, followed by vertically stretching the graph by a factor of 4?
   A) \( 4x^3 + 24 \)
   B) \( 4x^3 + 6 \)
   C) \( 24x^2 + 6 \)
   D) \( 4x^3 + 72x^2 + 432x + 864 \)

15. What is the equation for the graph obtained by shifting the graph of \( y = x^3 \) vertically upward by 3 units, followed by vertically stretching the graph by a factor of 5, followed by reflecting the graph across the x-axis?
   A) \( -5x^3 - 15 \)
   B) \( -5x^3 - 3 \)
   C) \( 15x^2 + 3 \)
   D) \( -5x^3 - 45x^2 - 135x - 135 \)

16. If the graph of \( y = f(x) \) is shrunk vertically by a factor of 1/2, then shifted vertically by 8 units, then stretched vertically by a factor of 4, the resulting graph is the same as the original graph.
   A) True
   B) False

17. Which function represents the following situation: the gravitational force, \( F \), between two bodies is inversely proportional to the square of the distance, \( d \), between them?
   A) \( F = kd^2 \)
   B) \( F = k\left(\frac{1}{d^2}\right) \)
   C) \( F^2 = k\left(\frac{1}{d}\right) \)
   D) \( F^2 = kd \)

18. A ball is thrown at time \( t=0 \) and its height above ground (in feet) \( t \) seconds after it is thrown is given by \( f(t) = -16t^2 + 96t + 6 \). Use a graphing calculator to graph \( f(t) \), and use the graph to answer the following.
   A. How high does the ball go?
   B. How long it it in the air?

19. Assume that the function shown in the following table is periodic. What is \( f(12) \)?

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
<td>-2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>
20. In nature, the populations of two animals, one of which preys upon the other (such as foxes and rabbits) are observed to oscillate with time, and are found to be well approximated by trigonometric functions. The population of foxes is shown in the graph below. How many foxes will there be in 21 months?

![Graph of Fox Population Over Time]

21. If \( k(t) = A \sin B\pi t + C \) defines the function graphed in the following figure, then \( A=\), \( B=\), and \( C=\).

![Graph of General Function]

22. What value of \( B \) would you use if \( \sin Bt \) was to model a periodic function with period 1 year where \( t \) is measured in months? Round to 4 decimal places.

23. Give a formula for the following sinusoidal function as a transformation of \( f(t) = \sin(t) \).

![Graph of Sinusoidal Function]

\( f(t) = -2 \sin 3t \)

\( f(t) = -2 \sin \left( \frac{t}{3} \right) \)

\( f(t) = 2 \sin 3t \)

\( f(t) = 2 \sin \left( \frac{t}{3} \right) \)

24. Temperature in Town A oscillates daily between \( 30^\circ \) F at 4am and \( 60^\circ \) F at 4pm. If \( H = A \cos(B\pi(t + C)) + D \) is a formula for the temperature in Town A in terms of time, where time is measured in hours from 8am, then \( A=\), \( B=\), \( C=\), and \( D=\).
25. Given the following data about the function \( f \), give the average rate of change of \( f \) between \( x = 3.0 \) and \( x = 3.8 \). Round to 2 decimal places.

\[
\begin{array}{cccc}
  x & 3.0 & 3.2 & 3.4 & 3.6 \\
  f(x) & 8.2 & 9.5 & 10.5 & 11.0 \\
\end{array}
\]

26. The height of an object in feet above the ground is given in the following table. The average velocity over the interval \( 0 \leq t \leq 3 \) is _____ feet/sec.

\[
\begin{array}{cccccc}
  t \text{(sec)} & 0 & 1 & 2 & 3 & 4 \\
  y \text{(feet)} & 10 & 45 & 70 & 85 & 90 \\
\end{array}
\]

27. Consider the two functions shown below.

A. 

B. 

A) The function in graph A is the derivative of the function in graph B.
B) The function in graph B is the derivative of the function in graph A.
C) Neither function is the derivative of the other.

28. Let \( f(T) \) be the time, in minutes, that it takes for an oven to heat up to \( T \)°F. What is the sign of \( f'(T) \)?

A) positive
B) negative

29. Every day the Undergraduate Office of Admissions receives inquiries from eager high school students (e.g. "Please send me an application", etc.) They keep a running count of the number of inquiries received each day, along with the total number received until that point. Below is a table of weekly figures from about the end of August to about the end of October of a recent year. Based on the table, find a formula for the total number of inquiries in a given week and use it to approximate the number of inquiries the admissions office will receive over the entire year. Round to the nearest 1000.

<table>
<thead>
<tr>
<th>Week of</th>
<th>Inquiries That Week</th>
<th>Total for Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/28–9/01</td>
<td>1085</td>
<td>11,928</td>
</tr>
<tr>
<td>9/04–9/08</td>
<td>1193</td>
<td>13,121</td>
</tr>
<tr>
<td>9/11–9/15</td>
<td>1312</td>
<td>14,433</td>
</tr>
<tr>
<td>9/18–9/22</td>
<td>1443</td>
<td>15,876</td>
</tr>
<tr>
<td>9/25–9/29</td>
<td>1588</td>
<td>17,464</td>
</tr>
<tr>
<td>10/02–10/06</td>
<td>1746</td>
<td>19,210</td>
</tr>
<tr>
<td>10/09–10/13</td>
<td>1921</td>
<td>21,131</td>
</tr>
<tr>
<td>10/16–10/20</td>
<td>2113</td>
<td>23,244</td>
</tr>
<tr>
<td>10/23–10/27</td>
<td>2325</td>
<td>25,569</td>
</tr>
</tbody>
</table>
30. Let \( L(r) \) be the amount of lumber, in board-feet, produced from a tree of radius \( r \) (measured in inches). Which of the following gives the rate of change in the amount of lumber, in board-feet per inch, with respect to the radius when the radius is 8 inches?
A) \( L(8) \)
B) \( L'(8) \)
C) \( r \) such that \( L(r) = 8 \)
D) \( r \) such that \( L'(r) = 8 \)

31. Suppose the graph of \( f \) is in the figure below. Is \( f(D) \) positive, negative, or zero?

32. Suppose the graph of \( f \) is in the figure below. Is \( f'(B) \) positive, negative, or zero?

33. Suppose the graph of \( f \) is in the figure below. Is \( f''(B) \) positive, negative, or zero?
34. A company graphs $C'(t)$, the derivative of the number of pints of ice cream sold over the past ten years. Out of $t=1,2,4,8,$ and 10, in what year was $C''(t)$ greatest?

35. If $f(x) = x^3 + 9x^2 - 3x + 3$, find $\frac{d^3y}{dx^3}$

36. Find the first derivative of $y^3 + \frac{7}{y}$.

A) $3y^2 + \frac{7}{y^2}$
B) $3y^2 - \frac{7}{y^2}$
C) $3y^2 + 7$
D) $3y^2 - 7$

37. Find the first derivative of $t = z^2 + \frac{6}{z} + 7z^{-2}$.

A) $2z + 6 \frac{-14}{z^3}$
B) $2z + 6 \frac{14}{z^3}$
C) $2z - 6 \frac{14}{z^3}$
D) $2z + 6 \frac{-14}{z}$

38. Find the first derivative of $s = t^\pi + \sqrt{3t}$.

A) $t^{2\pi} + \frac{\sqrt{3}}{2}$
B) $\sqrt{3}$
C) $\pi t^{\pi - 1} - \frac{1}{\sqrt{3}}$
D) $\pi t^{\pi - 1} + \sqrt{3}$
39. Find the first derivative of \( y = x^2 + 5x^3 - \frac{1}{x} \).
   A) \( 15x^2 + 2x + x^{-2} \)
   B) \( 15x^2 + 2x - 1 \)
   C) \( 15x^2 + 2x - x^{-2} \)
   D) \( 15x^2 + 2x + 1 \)

40. Find the first derivative of \( t = z^3 + \frac{5}{z^2} + 6z^{-3} \).
   A) \( 3z^2 + \frac{10}{z^3} + \frac{18}{z^4} \)
   B) \( 3z^2 - \frac{10}{z} - \frac{18}{z^3} \)
   C) \( 3z^2 + \frac{5}{z} - \frac{18}{z^4} \)
   D) \( 3z^2 + \frac{5}{z} - \frac{18}{z^4} \)

41. Find the first derivative of \( w = cx^2 + x^4 \)
   A) \( x^2 + tx^4 \)
   B) \( 2cx + tx \)
   C) \( 2cx + t \)
   D) \( 2cx + tx^4 \)

42. The equation for the tangent line to the curve \( x^2 + y = 4 \) when \( x = 1 \) is \( y = \ldots \) \( x + \ldots \).

43. The curve \( f(x) = x^4 - 49x^2 + 3 \) has a horizontal tangent at which of the following points?
   A) \( 1 \)
   B) \( -1 \)
   C) \( \frac{7\sqrt{2}}{2} \)
   D) \( -\frac{7\sqrt{2}}{2} \)
   E) \( 0 \)

44. Find the derivative of \( f(x) = 10\sqrt{x} + x^3 \).
   A) \( -\frac{5}{\sqrt{x}} + 3x^2 \)
   B) \( \frac{5}{\sqrt{x}} + 3x^2 \)
   C) \( 5\sqrt{x} + 3x^2 \)
   D) \( 10 + 3x^2 \)

45. Let \( g(t) = 6t^3 - t^2 + 2t \). Find \( g'(t) \).

46. Consider the function \( f(x) = 5x^4 - 6x^3 + 8 \). We know that \( f(x) \) is concave down when \( \ldots < x < \ldots \).
47. Consider the graphs of \( s = \ln(t) \) and \( t = e^x \). Find the slope of the first curve when \( t = 4 \).

48. Find the equation of the tangent line to the curve \( y = e^x \) which passes through the origin.

49. Find the derivative of \( 9^x - 9 \).
   A) \( \ln(9)9^x \)
   B) \( \ln(9)9^{x-1} \)
   C) \( x9^{x-1} \)
   D) \( \ln(8)8^x \)

50. Find the derivative of \( f(x) = 8e^x - 8^x \).
   A) \( \ln(8)e^x - \ln(8)8^x \)
   B) \( 8e^x - \ln(8)8^x \)
   C) \( 8e^x - x8^{x-1} \)
   D) \( 8xe^{x-1} - x8^{x-1} \)

51. Find the derivative of \( g(x) = 8e^{\pi x} \).
   A) \( 8e^{\pi x} \)
   B) \( 8\pi xe^{\pi x-1} \)
   C) \( 8\pi e^{\pi x} \)
   D) \( 8\ln(\pi)e^{\pi x} \)

52. Find the derivative of \( f(x) = (\ln 8)x^2 + (\ln 8)e^x \)
   A) \( 2(\ln 8)x + (\ln 8)e^x \)
   B) \( (\ln 16)x + (\ln 8)e^x \)
   C) \( 2(\ln 8)x + (\ln 8)xe^{x-1} \)
   D) \( (\ln 16)x + (\ln 8)xe^{x-1} \)

53. Find the derivative of \( g(x) = 5x - \frac{1}{\sqrt[5]{x}} + 5^x \)
   A) \( 6 + x(5^{x-1}) \)
   B) \( 5 + \frac{1}{5x^{\frac{6}{5}}} + \ln 5(5^x) \)
   C) \( 5 + \frac{5}{x^{\frac{6}{5}}} + \ln 5(5^x) \)
   D) \( 5 + \frac{1}{5x^{\frac{1}{5}}} + x(5^{x-1}) \)
54. Find the derivative of \( h(t) = t^{\pi^5} + (\pi^5)^t + \pi t^5 \).
   A) \( \ln(t) t^{\pi^5} + \ln(\pi^5)(\pi^5)^t + 5\pi t^4 \)
   B) \( \pi^5 t^{(\pi^5 - 1)} + t(\pi^5)^t - 1 + 5\pi t^4 \)
   C) \( \pi^5 t^{(\pi^5 - 1)} + \ln(\pi^5)(\pi^5)^t + 5\pi t^4 \)
   D) \( 5\pi^4 t^{(\pi^5 - 1)} + \ln(\pi^5)(\pi^5)^t + 5\pi t^4 + t^5 \)

55. Find the derivative of \( g(t) = (4/e)^t + e^t + 2 \).
   A) \( t(4/e)^{t-1} + te^{t-1} \)
   B) \( -(4/e)^{2t} + e^t \)
   C) \( (4/e)^t + e^t \)
   D) \( -(4/e)^t + e^t \)

56. Find the first derivative of \( s = \ln z^6 \).
   A) \( 1/z^6 \)
   B) \( 6/z \)
   C) \( 6/z^6 \)
   D) \( 6\ln z \)

57. Find the first derivative of \( y = e^{-x^2} \).
   A) \( -2xe^{-x^2} \)
   B) \( e^{-x^2} \)
   C) \( -x^2e^{-x^2} \)
   D) \( -2x + e^{-x^2} \)

58. Find the derivative of \( g(x) = \sqrt{4x^3 + e^x} \).
   A) \( 1/2\sqrt{4x^3 + e^x} \)
   B) \( 12x^3 + e^x \)
   C) \( 2\sqrt{4x^3 + e^x} \)
   D) \( 1/2\sqrt{12x^2 + e^x} (12x^2 + e^x) \)

59. Consider the function \( g(x) = (ax^2 + b)^2 \), where \( a \) and \( b \) are constants. Find \( g''(x) \).

60. The population of Mexico in millions is described by the formula \( P(t) = 67e^{0.027t} \), where \( t \) is the number of years after 1980. How many years will it take for the population to triple? Round to 2 decimal places.
61. \( P(t) = (90,000)e^{0.05t} \), where \( t \) is the number of years after 1990. How many years will it take for the house to triple in value? Round to the nearest year.

62. What is \( \frac{d}{dx}(6^x \cdot \ln x) \)?
   A) \( \ln 6 \cdot 6^x \ln x + \frac{6^x}{x} \)
   B) \( \ln 6 \cdot 6^x + \frac{1}{x} \)
   C) \( \ln 6 \cdot 6^x \cdot \frac{1}{x} \)
   D) \( x6^{x-1} \cdot \ln x + \frac{6^x}{x} \)

63. Compute the derivative of \( y = 5x^4 + \frac{x^2 - 9x}{x^3} \).
   A) \( 20x^3 + 18x^{-3} + x^{-2} \)
   B) \( 20x^3 + 18x^{-3} - x^{-2} \)
   C) \( 20x^3 - 18x^{-3} + x^{-2} \)
   D) \( 20x^3 - 18x^{-3} - x^{-2} \)

64. The following table gives values for two functions \( f \) and \( g \) and their derivatives. What is \( \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)_{x=1} \)? Round to 2 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( g )</td>
<td>1</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>( f' )</td>
<td>-3</td>
<td>-2</td>
<td>-1.5</td>
<td>-1</td>
</tr>
<tr>
<td>( g' )</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

65. Differentiate \( \frac{7x^2}{x^3 + 1} \).
   A) \( \frac{14}{3x} \)
   B) \( \frac{7x(x^3 - 2)}{(x^3 + 1)^2} \)
   C) \( \frac{7x(-x^3 + 2)}{(x^3 + 1)^2} \)
   D) \( \frac{7x}{(x^3 + 1)} \)
66. Find \( \frac{dy}{dx} \) if \( y = 6x^2 \sin(\pi x) \).

A) \( 12\pi x \cos(\pi x) \)
B) \( 12x \sin(\pi x) + 6\pi x^2 \cos(\pi x) \)
C) \( 12x \sin(\pi x) - 6\pi x^2 \cos(\pi x) \)
D) \( 12x \sin(\pi x) + 6x^2 \cos(\pi x) \)

67. Find the derivative of \( y = 9 \sin(3^{-x}) \).

A) \( 9 \ln 3 \cdot 3^{-x} \cdot \cos(3^{-x}) \)
B) \( -9 \ln 3 \cdot 3^{-x} \cdot \cos(3^{-x}) \)
C) \( -9x \cdot 3^{-x-1} \cdot \cos(3^{-x}) \)
D) \( 9 \cos(3^{-x}) \)

68. Find the derivative of \( f(w) = \sin(2w^2) + \cos(2w^2) \).

A) \( 4w \cos(2w^2) - 4w \sin(2w^2) \)
B) \( 4w \sin(2w^2) - 4w \cos(2w^2) \)
C) \( 2 \cos(2w^2) - 2 \sin(2w^2) \)
D) \( \cos(2w^2) - \sin(2w^2) \)

69. Find the derivative of \( g(x) = \cos(\sin(7x)) \).

A) \( 7 \cos(7x) \sin(\sin(7x)) \)
B) \( -7 \cos(7x) \sin(\sin(7x)) \)
C) \( -7 \sin(\cos(7x)) \)
D) \( 7 \sin(\cos(7x)) \)

70. Find the derivative of \( h(z) = e^{\cos z} + e^{\sin z} \).

A) \( e^{\cos z} + e^{\sin z} \)
B) \( \cos ze^{\cos z} + \sin ze^{\sin z} \)
C) \( -\sin ze^{\cos z} + \cos ze^{\sin z} \)
D) \( \sin ze^{\cos z} - \cos ze^{\sin z} \)

71. Differentiate \( f(\theta) = \sin(\theta^b) \). Assume \( b \) is a positive constant.

A) \( b \cos(\theta^b) \)
B) \( \ln(\theta^b) \cos(\theta^b) \)
C) \( b \theta^{b-1} \cos(\theta^b) \)
D) \( b \cos(\theta^{b-1}) \)
72. The following figure is a graph of a derivative function, $f'$. Indicate on the graph the $x$-values that are critical points and label each as a local maximum, a local minimum, or neither.

![Graph of a derivative function](image)

73. A brick is heated in an oven and taken out to cool off after a certain time. The temperature $T$ of the brick at any time $t$ is given by $T = 100e^{-(t-1)^2}$ for $t \geq 0$, with $T$ in degrees Celsius and $t$ in minutes. What is the temperature of the brick when it is placed in the oven (to the nearest degree)?

74. Assume that the polynomial $f$ has exactly one local maximum, two local minima, and two inflection points. What is the smallest number of zeros $f$ could have?

75. If $f(x) = e^{-x} \cos x$ for $0 \leq x \leq 2\pi$, what is $f'(x)$?
   A) $-e^{-x} \cos x - e^{-x} \sin x$
   B) $-e^{-x} \cos x + e^{-x} \sin x$
   C) $-e^{-x} \sin x$
   D) $e^{-x} \sin x$

76. If $f(x) = e^{-x} \cos x$ for $0 \leq x \leq 2\pi$, which of the following are inflection points of $f(x)$?
   A) $0$
   B) $\frac{\pi}{4}$
   C) $\frac{3\pi}{4}$
   D) $\frac{\pi}{4}$
   E) $\frac{7\pi}{4}$

77. The point $x=1$ on the following closed graph corresponds to
   A) a local minimum
   B) a local maximum
   C) neither a maximum nor a minimum
   D) a local and global minimum
   E) a local and global maximum

![Closed graph with point at x=1](image)
78. The distance, $s$, traveled by a runner in a 20 mile race is given in the following figure, where time, $t$, is in hours. At which of the following values of $t$ is the runner’s speed is the slowest?

![Graph showing distance vs. time for a runner.]

A) 4  
B) 3  
C) 1  
D) 2

79. The quantity of a medication in the bloodstream $t$ hours after it is ingested is given, in mg, by $q(t) = 50te^{-t}$. What is the maximum quantity of the medication in the bloodstream?

A) 18 mg  
B) 50 mg  
C) 136 mg  
D) 25 mg

80. The following table shows cost and revenue for a product (in dollars).

<table>
<thead>
<tr>
<th>Q</th>
<th>R</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>100</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>2000</td>
<td>1500</td>
<td>800</td>
</tr>
<tr>
<td>3000</td>
<td>2000</td>
<td>1400</td>
</tr>
<tr>
<td>4000</td>
<td>2500</td>
<td>2000</td>
</tr>
</tbody>
</table>

A. What are the fixed costs?  
B. At what value of $q$ is profit is maximized?

81. What is the maximum profit obtainable if the total revenue and total cost (in dollars) are given by

$$R(x) = 6x - 0.001x^2$$
$$C(x) = 300 + 1.1x$$

Round to the nearest dollar.

82. What is the maximum profit obtainable if gadgets sell for $450 per unit and the total cost (in dollars) of producing $x$ units is given by $C(x) = 10,500 + 3x^2$?

83. Write a formula for total cost, $C$, as a function of quantity $r$ when fixed costs are $45,000 and and variable costs are $1,300 per item.

84. Find the quantity $q$ which maximizes profit if the total revenue, $R(q)$, and the total cost, $C(q)$, are given in dollars by $R(q) = 4q - 0.05q^2$ and $C(q) = 250 + 3q$, where $0 \leq q \leq 700$ units.
85. The rabbit population, \( P \), in a wilderness area is approximated by the function
\[
P = \frac{800}{1 + 11e^{-0.24t}},
\]
where \( t \) is the number of weeks since the rabbits were introduced into the area.

A. How many rabbits were initially introduced into the area (to the nearest rabbit)?
B. How many rabbits were in the area after 10 weeks (to the nearest rabbit)?
C. What is the carrying capacity of rabbits in the area?

86. The following table shows the total sales, in thousands, since a new DVD was released.

A. Estimate the point of diminishing returns.
B. Using your answer from part (A), predict the total possible sales for the DVD.

<table>
<thead>
<tr>
<th>week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>sales</td>
<td>0</td>
<td>9</td>
<td>24</td>
<td>60</td>
<td>141</td>
<td>294</td>
</tr>
</tbody>
</table>

87. A flu epidemic spreads amongst a group of people according to the formula
\[
P(t) = \frac{1400}{1 + 199e^{-0.9t}},
\]
where \( P(t) \) represents the number of people that are infected by the end of day \( t \).

A. How many people are infected by the end of the fifth day (to the nearest person)?
B. At what rate do the people become infected on day 5 (to the nearest person per day)?

88. The rate of pollution pouring into a lake is measured every 10 days, with results in the following table. About how many tons of pollution have entered the lake during the first 40 days?

<table>
<thead>
<tr>
<th>Time in days</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of pollution in tons/day</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

89. A car is observed to have the following velocities at times \( t = 0, 2, 4, 6 \):

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (ft/sec)</td>
<td>0</td>
<td>21</td>
<td>40</td>
</tr>
</tbody>
</table>

Give a lower estimate for the number of feet the car traveled in first 6 seconds.

90. At time \( t \), in seconds, the velocity \( v \), in miles per hour, of a car is given by
\[
v(t) = 5 + 0.5t^2 \quad \text{for} \quad 0 \leq t \leq 8.
\]
Use \( \Delta t = 2 \) to estimate how many miles were traveled during this time (average left- and right-hand sums).

91. Use a calculator to estimate
\[
\int_{0}^{4} (x^3 + 1) \, dx.
\]
Round your answer to 2 decimal places.
92. Use the following graph to estimate \( \int_{10}^{70} f(t) \, dt \) using three terms of the right Reimann sum.

![Graph](graph1.png)

93. Using the following figure, calculate the value of the left-hand Reimann sum for the function \( f \) on the interval \( 0 \leq t \leq 12 \) using \( \Delta t = 3 \).

![Graph](graph2.png)

94. Your rich eccentric friend has hired you to cover his back yard with grass and patio stone. Instead of giving you a map, he gives you the equations of the boundary lines. If the southwest corner of his yard is taken as the origin, with the \( x \)-axis pointing eastward and distances measured in feet, then the boundaries of the yard are the lines \( x = 0 \), \( x = 100 \), \( y = 0 \), and \( y = 110 - 0.5x \). The border between grass and stone is \( y = 40 + 20 \cos \left( \frac{\pi t}{40} \right) \), with grass covering all of the yard south of the curve. This border also bounds one side of the pool, the other side of the pool being surrounded by the curve \( y = 60 - 0.05(x - 40)^2 \). All of the rest of the yard is to be covered by stone. Draw a sketch of the yard, and use it to estimate, to the nearest square yard, the area of the grass.
95. A car is moving along a straight road from \( A \) to \( B \), starting from \( A \) at time \( t = 0 \). Below is the velocity (positive direction is from \( A \) to \( B \)) plotted against time. How many kilometers away from \( A \) is the car at time \( t = 6 \)?

![velocity plot](image)

96. Estimate the area of the region under the curve \( y = \sin(x/2) \) for \( 0 \leq x \leq 3 \). Round to 2 decimal places.

97. The Ethnic food line at the Cougar Eat can serve customers at the rate of about 25 per hour. From 10 am until 4 pm one day, the rate \( R \) at which customers entered the line was about \( R(t) = 45 - 5(t - 3)^2 \) customers per hour at \( t \) hours past 10 am.

About what time did a waiting line form?
A) 11:00 am  
B) 11:30 am  
C) 11:15 am  
D) 11:45 am

98. The Ethnic food line at the Cougar Eat can serve customers at the rate of about 25 per hour. From 10 am until 4 pm one day, the rate \( R \) at which customers entered the line was about \( R(t) = 45 - 5(t - 3)^2 \) customers per hour at \( t \) hours past 10 am.

About when was the waiting line the longest?
A) 2:30 pm  
B) 3:00 pm  
C) 2:45 pm  
D) 2:15 pm

99. The Ethnic food line at the Cougar Eat can serve customers at the rate of about 30 per hour. From 10 am until 4 pm one day, the rate \( R \) at which customers entered the line was about \( R(t) = 45 - 5(t - 3)^2 \) customers per hour at \( t \) hours past 10 am.

About how many customers were served between 10 am and 4 pm that day? Round to the nearest whole number.

100. If the velocity function \( v(t) \) is measured in feet per second and \( t \) gives time in seconds, what are the units of measurement for \( \int_0^2 v(t) \, dt \)?

A) feet  
B) feet/second  
C) feet/second^2  
D) seconds/foot
101. The graph of \( f'(x) \) is shown in the following figure. Given that \( f(0) = 10 \), find \( f(40) \).

![Graph of f'(x)](image)

102. If \( f'(t) = 2t \) is a production rate, measured in items per hour, then how many items were produced from hour 2 to hour 6?

103. What is the antiderivative of \( f(x) = 3x^2 + 2 \)?
   A) \( x^3 + 2x + C \)
   B) \( 3x^3 + 2x + C \)
   C) \( 6x + 2 + C \)
   D) \( 6x + C \)

104. What is the antiderivative of \( h(t) = \frac{8}{t} \)?
   A) \( -\frac{8}{t} + C \)
   B) \( 8\ln|t| + C \)
   C) \( 8t\ln|t| + C \)
   D) \( \frac{8t}{\ln|t|} + C \)

105. Find an antiderivative \( F(x) \) of \( f(x) = e^x + 1 \) such that \( F(0) = 2 \).
   A) \( e^x + x + 1 \)
   B) \( e^x + x + 2 \)
   C) \( e^x + x \)
   D) \( xe^x + x \)

106. Evaluate \( \int (x^2 + 4x - 4)\,dx \).
   A) \( \frac{x^3}{3} + 2x^2 - 4x + C \)
   B) \( x^3 + 2x^2 - 4x + C \)
   C) \( \frac{x^3}{3} + 4x^2 - 4x + C \)
   D) \( x^3 + 4x^2 - 4x + C \)
107. Find the indefinite integral \( \int e^{5t} \, dt \).
   
   A) \( \frac{1}{6} e^{5t} + C \)
   
   B) \( e^{5t} + C \)
   
   C) \( \frac{1}{5} e^{5t} + C \)
   
   D) \( \frac{1}{5t+1} e^{5t} + C \)

108. \( \int \frac{3}{\sqrt[3]{x}} \, dx = 6\sqrt{x} + C \)
   
   A) True
   
   B) False

109. \( \int (t^2 + \frac{4}{t^2}) \, dt = \frac{t^3}{3} - \frac{4}{t} + C \)
   
   A) True
   
   B) False

110. Find the antiderivative of \( f(x) = \frac{a}{x} + b \), where \( a \) and \( b \) are constants.

   A) \( \frac{2a}{x^2} + bx \)

   B) \( \frac{a}{x^2} + bx \)

   C) \( a \ln|x| + bx \)

   D) \( \frac{1}{a} \ln|x| + bx \)

111. Find \( \int t^4 \sin(t^5) \, dt \) using integration by substitution.

   A) \( \frac{1}{5} \cos(t^5) + C \)

   B) \( -\frac{1}{5} \cos(t^5) + C \)

   C) \( \frac{1}{5} t^5 \cos(t^5) + C \)

   D) \( -\frac{1}{5} t^5 \cos(t^5) + C \)
112. Find \( \int xe^{4x^2} \, dx \) using integration by substitution.

A) \( \frac{1}{8} e^{4x^2} + C \)
B) \( \frac{1}{4} e^{4x^2} + C \)
C) \( \frac{1}{3} e^{4x^2} + C \)
D) \( \frac{1}{12} e^{4x^2} + C \)

113. Which of the following is equivalent to \( f(x) = (x^2 + 1)^7 + C \)?

A) \( \int 14x(x^2 + 1)^6 \, dx \)
B) \( \int 7x(x^2 + 1)^6 \, dx \)
C) \( \int 14x(x^2 + 1)^8 \, dx \)
D) \( \int 7x(x^2 + 1)^8 \, dx \)

114. Evaluate \( \int 12x^2 \, dx \).

115. Evaluate \( \int (x^2 + 4x + 4) \, dx \).

116. Evaluate \( \int \frac{2}{x^3} \, dx \). Round to 2 decimal places.

117. Propellant is leaking out from the pressurized fuel tanks of the space shuttle at a rate of \( r(t) = 15e^{-0.1t} \) psi per second at time \( t \) in seconds. How many psi have leaked during the first 1 1/2 minutes? Round to 2 decimal places.

118. At time \( t \) hours after taking medication, the rate at which the medication is being eliminated from the body is given by \( r(t) = 30(e^{-0.2t} - e^{-0.3t}) \) mg/hr. Assuming that all of the medication is eventually eliminated, how many mg was the original dose?

119. The following figure is a graph of \( f'(x) \). On which of the following intervals is \( f \) increasing?

A) \( 1 < x < 3 \)
B) \( -2 < x < 1 \)
120. Given the following graph of $g'(x)$ and the fact that $g(0) = 2000$, determine whether $g''(150)$ is positive or negative.

121. Given the following graph of $g'(x)$ and the fact that $g(0) = 2000$, what is $x = 300$?

A) a local minimum
B) a local maximum
C) an inflection point
D) none of the above
122. The following graph represents the rate of change of a function $f$ with respect to $x$, i.e., it is the graph of $f'$, with $f(0) = 0$. Which of the following are true at $x = 1.3$?

A) $f$ is concave down  
B) $f$ is concave up  
C) $f$ is increasing  
D) $f$ is decreasing

123. The following figure shows the graph of $f$. If $F' = f$ and $F(0) = 0$, find $F(4)$.

124. Which of the following graphs best describes the speed of a car merging onto the freeway?
125. A population of rodents grows at a rate proportional to the size of the population. Which of the following is the differential equation for the size of the population, \( P \), as a function of time, \( t \)?

A) \[
\frac{dP}{dt} = kP, \text{ with } k \text{ positive}
\]

B) \[
\frac{dP}{dt} = kP, \text{ with } k \text{ negative}
\]

C) \[
\frac{dP}{dt} = \frac{k}{P}, \text{ with } k \text{ positive}
\]

D) \[
\frac{dP}{dt} = \frac{k}{P}, \text{ with } k \text{ negative}
\]

126. A person withdraws money from a trust fund at a rate of $13,000 per year, and the account is earning interest at a rate of 5% per year, compounded continuously. Write a differential equation for the balance, \( B \), in the account as a function of time, \( t \), in years and use it to calculate \( \frac{dB}{dt} \) if \( B = $100,000 \).

127. A drug is administered intravenously to a patient at a rate of 14 mg per day. About 40% of the drug in the patient's body is metabolized and leaves the body each day. Which is the differential equation for the amount of the drug, \( D \), in the body as a function of time, \( t \), in days?

A) \[
\frac{dD}{dt} = 14 - 40D
\]

B) \[
\frac{dD}{dt} = 14 - 0.4D
\]

C) \[
\frac{dD}{dt} = 40 - 14D
\]

D) \[
\frac{dD}{dt} = 0.4 - 14D
\]

128. Which of the following are solutions to the differential equation \( \frac{dy}{dx} = 3x \)?

A) \( y = 3x \)

B) \( y = e^{3x} \)

C) \( y = x \)

D) \( y = 3e^x \)

E) \( y = x^3 \)

F) none of the above

129. Which of the following are solutions to the differential equation \( \frac{dy}{dx} = 3 \frac{y}{x} \)?

A) \( y = 3x \)

B) \( y = e^{3x} \)

C) \( y = x \)

D) \( y = 3e^x \)

E) \( y = x^3 \)
130. What is the solution of \( \frac{dP}{dt} = -12e^{-t} \) when \( P(0) = 17 \)?

A) \( P = 12e^{-t} - 5 \)
B) \( P = 12e^{-t} + 5 \)
C) \( P = -12e^{-t} + 29 \)
D) \( P = -12e^{-t} + 29 \)

131. If \( y = x^2 + k \) is a solution to the differential equation \( 2y - x \frac{dy}{dx} = 8 \), then \( k = \) _____.

132. The solution to the differential equation \( \frac{dy}{dx} - \frac{y}{4} = 0 \) subject to the initial condition \( y(1) = 29 \) is \( y = Ce^{kx} \), where \( C = \) _____ and \( k = \) _____. Round answers to 2 decimal places.

133. A radioactive isotope decays at a continuous rate of approximately 25% per day. If \( A \) is the amount of the isotope and \( t \) is time in days, what is the differential equation for this situation and its general solution?

A) \( \frac{dA}{dt} = -0.25A; \quad A = Ce^{-0.25t} \)
B) \( \frac{dA}{dt} = -0.75A; \quad A = Ce^{-0.75t} \)
C) \( \frac{dA}{dt} = 0.25A; \quad A = Ce^{0.25t} \)
D) \( \frac{dA}{dt} = 0.75A; \quad A = Ce^{0.75t} \)

134. The amount of medicine present in the blood of a patient decreases due to metabolism according to the exponential decay model. One hour after a dose was given, there were 3.7 ng/cm\(^3\) present, and a hour later there were 2.5 ng/cm\(^3\). After how many hours will there be less than 0.5 ng/cm\(^3\) present, assuming no more medication is taken? Round to 1 decimal place.

135. If no more pollutants are dumped into a lake, the amount of pollution in the lake will decrease at a rate proportional to the amount of pollution present. If there are 400 units of pollution present initially and 184 units left after 8 years, use differential equations to find the number of units left after 12 years. Round to 1 decimal place.
Answer Key

1. \( N = -5,600 + 780t \)
2. A. III and IV  
   B. II and III  
   C. I  
   D. I
3. A
4. D
5. C
6. A
7. A
8. C
9. A
10. 72%
11. 1.232
12. IV
13. 36.06
14. A
15. A
16. B
17. B
18A. A. 150 feet  
   18B. B. 6 seconds
19. -4
20. 700
21A. 4
21B. 1/5
21C. 2
22. 0.5236
23. D
24A. -15
24B. 1/12
24C. 4
24D. 45
25. 6.25
26. 25
27. A
28. A
29. 66,000
30. B
31. positive
32. positive
33. negative
34. 1
35. 6
36. B
37. C
38. D
39. A
40. B
41. D
42A. -2
42B. 5
43. C, D, E
44. B
45. \( 18t^2 - 2t + 2 \)
46A. 0
46B. 0.6
47. 0.25
48. \( y = ex \)
49. A
50. B
51. C
52. A
53. B
54. C
55. D
56. B
57. A
58. B
59. \( 12a^2x^2 + 4ab \)
60. 40.69
61. 22
62. A
63. B
64. -0.92
65. C
66. B
67. B
68. A
69. B
70. C
71. C
72. 37
73. 37
74. 0
75. A
76. D
77. A
78. B
79. A
80A. A. $100  
   80B. B. $3000  
   81. $5702  
   82. $6,375  
   83. \( C = 45,000 + 1,300r \)
84. 10
85. A. 67
   B. 400
   C. 800
86. A. 506,000  
   B. 1,012,000
87. A. 436  
   B. 270  
   88. 325
   89. 122
   90. 128
   91. 1649.25
   92. 16,000
   93. 150
   94. 473
   95. 9
   96. 1.86
   97. A
   98. B
   99. 163
   100. A
   101. 25
   102. 32
   103. A
   104. B
   105. A
   106. A
   107. C
   108. A
   109. A
   110. C
   111. B
   112. A
   113. A
   114. 104
   115. 39
   116. 0.89
   117. 149.98
   118. 50
   119. A
   120. negative
   121. C
   122. A, D
   123. 0
   124. II
   125. A
   126. -$8000
   127. B
   128. F
   129. E
   130. B
   131. 4
   132A. 0.25
   132B. 22.59
   133. A
   134. 6.1
   135. 124.9