Find the locations of all absolute extrema if they exist.
1) \( f(x) = -3x^4 + 20x^3 - 36x^2 + 2 \)

Evaluate the definite integral.
2) \( \int_0^1 \frac{4x^3}{(1 + x^4)^4} \, dx \)
3) \( \int_2^5 \frac{dt}{1 + t} \)
4) \( \int_0^1 \frac{5x^2}{8} \, dx \)

Find the area between the curves.
5) \( y = 2x - x^2, \quad y = 2x - 4 \)

Find the largest open intervals where the function is concave upward.
6) \( f(x) = 4x^3 - 45x^2 + 150x \)

Find the partial derivative.
7) Let \( z = f(x, y) = x^3 - 7x^2y + 2xy^3 \). Find \( \frac{\partial z}{\partial x} \).

Find the integral.
8) \( \int x^6 \, dx \)
9) \( \int e^{2x} x^2 \, dx \)
10) \( \int 8e^{4y} \, dy \)
11) \( \int (1 - 6x)e^{3x - 9x^2} \, dx \)
12) \( \int \frac{x}{(7x^2 + 3)^5} \, dx \)
13) \( \int \left( \frac{5}{x^2} - \frac{4}{\sqrt{x}} \right) \, dx \)
14) \( \int (7x^2 - 3x) \, dx \)
15) \( \int \frac{3e^{\sqrt{x}}}{8\sqrt{x}} \, dz \)
16) \( \int \frac{x^2 + 12x}{(x + 6)^2} \, dx \)

Find the open interval(s) where the function is changing as requested.
17) Decreasing; \( f(x) = x^3 - 4x \)
18) Increasing; \( f(x) = \frac{1}{x^2 + 1} \)

Find the derivative.
19) \( f(x) = 20x^{1/2} - \frac{1}{2}x^{20} \), find \( f'(x) \)
20) \( f(x) = (x^3 - 8)^{2/3} \)
21) \( y = 4e^{x^2} \)
22) \( y = (2x - 1)^3(x + 7)^{-3} \)

Find \( f_x(x, y) \).
23) \( f(x, y) = \ln(xy) \)

Decide if the given value of \( x \) is a critical number for \( f \), and if so, decide whether the point for \( x \) on \( f \) is a relative minimum, relative maximum, or neither.
24) \( f(x) = x^5; \quad x = 0 \)

Find the derivative of the given function.
25) \( (x^2 + 4)^3 \)

Use the properties of limits to help decide whether the limit exists. If the limit exists, find its value.
26) \( \lim_{x \to 4} \frac{x^2 + 6x - 40}{x^2 - 16} \)

Find \( f''(x) \) for the function.
27) \( f(x) = 8e^{-x^2} \)

Find all values of \( x \) (if any) where the tangent line to the graph of the function is horizontal.
28) \( y = x^3 - 12x + 2 \)
Find all points where the function is discontinuous.

29)

Find the location of the indicated absolute extremum within the specified domain.

30) Minimum of \( f(x) = (x^2 + 4)^{2/3}; \ [-2, 2] \)

Find the derivative of the function.

31) \( y = \ln (4 + x^2) \)

Solve the problem.

32) One airplane is approaching an airport from the north at 169 km/hr. A second airplane approaches from the east at 184 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 28 km away from the airport and the westbound plane is 23 km from the airport.

33) An architect needs to design a rectangular room with an area of 60 ft\(^2\). What dimensions should he use in order to minimize the perimeter?

Find the values of any relative extrema.

34) \( f(x) = \frac{x^2 + 1}{x^2} \)

Give an appropriate answer.

35) Find the instantaneous rate of change for the function \( f(x) = 5x^2 + x \) at \( x = -4 \).

Evaluate \( dy/dt \) for the function at the point.

36) \( xy^2 = 4; \ dx/dt = -5, x = 4, y = 1 \)

The graphs of a function \( f(x) \) and its derivative \( f'(x) \) are shown below. Decide which is the graph of \( f(x) \) and which is the graph of \( f'(x) \).

37)

Find dy/dx by implicit differentiation.

38) \( xy + x = 2 \)

Find all points where the function has any relative extrema or saddle points and identify the type of relative extremum.

39) \( f(x,y) = x^3 - 12xy + 8y^3 \)
1) Absolute maximum at \( x = 0 \); no absolute minima

2) \( \frac{7}{24} \)

3) \( \ln 2 \)

4) \( \frac{5}{24} \)

5) \( \frac{32}{3} \)

6) \( \left[ \frac{15}{4}, \infty \right) \)

7) \( 3x^2 - 14xy + 2y^3 \)

8) \( \frac{x^7}{7} + C \)

9) \( \frac{1}{2}x^2e^{2x} - \frac{1}{2}x^2e^{2x} + \frac{1}{4}e^{2x} + C \)

10) \( 2e^{4y} + C \)

11) \( \frac{1}{3}e^{3x} - 9x^2 + C \)

12) \( \frac{-1}{56(7x^2 + 3)^4} + C \)

13) \( -\frac{5}{x} - 8\sqrt{x} + C \)

14) \( \frac{7}{3}x^3 - \frac{3}{2}x^2 + C \)

15) \( \frac{3}{4}e^{\sqrt{2}} + C \)

16) \( x + \frac{36}{x + 6} + C \)

17) \( \left[ -\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3} \right] \)

18) \( (-\infty, 0) \)

19) \( 10x - \frac{1}{2} - 10x^{19} \)

20) \( f(x) = \frac{2x^2}{3} \sqrt{x^3 - 8} \)

21) \( 8xe^{x^2} \)

22) \( \frac{dy}{dx} = 45(2x - 1)^2(x + 7)^{-4} \)

23) \( \frac{y}{x} \)

24) Critical number but not an extreme point

25) \( 6x^5 + 48x^3 + 96x \)

26) \( \frac{7}{4} \)

27) \( 32x^2 e^{-x^2} - 16 e^{-x^2} \)

28) 2, -2

29) None

30) \( x = 0 \)

31) \( \frac{-2x}{x^2 + 4} \)

32) 353 km/hr

33) 7.75 ft \times 7.75 ft

34) No relative extrema.

35) -39

36) \( \frac{5}{8} \)

37) \( f(x) \) is the dashed line; \( f(x) \) is the solid line.

38) \( \frac{-1 + y}{x} \)

39) Relative minimum at (2,1) and saddle point at (0,0)