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## Rational Functions Worksheet

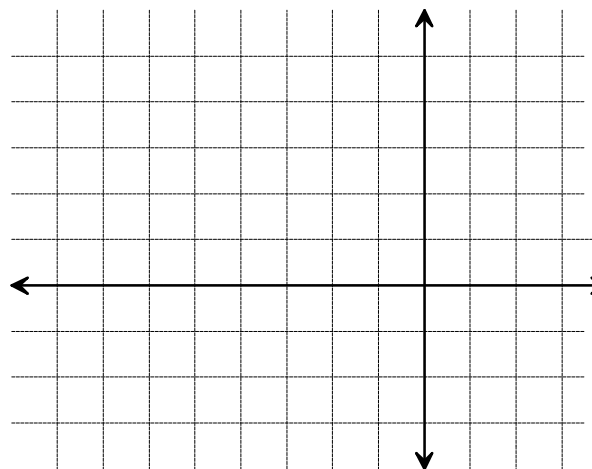
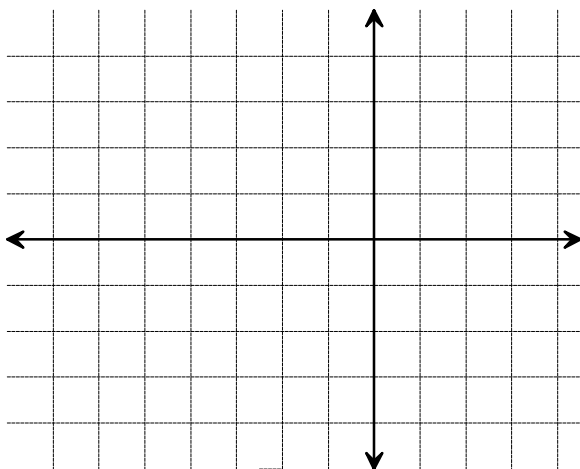
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Use the rules you have learned for  $f(x) = \frac{a_0 x^m \dots}{b_0 x^n \dots}$

Use your graphing calculator to help you to draw the following graphs on this worksheet include your work in you homework. For each problem be sure to note the asymptotes, domain, range, x intercepts, y intercepts, name any holes in the graph. Working these problems will help you learn the differences between the four different cases of rational functions. It will also help you understand the kinds of questions you will be asked about rational functions. Draw complete graphs

1.  $y = \frac{1}{x+3}$   $m < n$  so  $y = 0$  is the horizontal asymptote;  $x = -3$  is a vertical asymptote.  
 domain is  $x \in \mathbb{R}_e, x \neq -3$  ;  
 range is  $x \in \mathbb{R}_e, y \neq 0$  ;  
 one-to-one function ;  
 there is no x intercept;  
 the y intercept is  $(0, \frac{1}{3})$

2.  $y = \frac{2x}{x+3}$   $m = n$  so  $y = \frac{a_0}{b_0}$  is the horizontal asymptote, so it is  $y = 2$ ;  
 $x = -3$  is the vertical asymptote;  
 domain is  $x \in \mathbb{R}_e, x \neq -3$  ;  
 range is  $y \in \mathbb{R}_e, y \neq 2$  ;  
 the x & y intercept is at the origin  $(0, 0)$  .



3.  $f(x) = \frac{x^2 + 2x - 3}{x+3}$   $x \neq -3$

$m > n$  , the denominator divides evenly so there is a hole in the graph.

$$y = \frac{(x+3)(x-1)}{x+3}$$

$$y = \frac{(x+3)(x-1)}{x+3}$$

$$y = x - 1$$

In  $g(x) = x - 1$  let  $x = -3$  to find the hole.

$g(-3) = -3 - 1 = -4$  so hole is at  $(-3, -4)$ ;

domain  $x \in \mathbb{R}_e, x \neq -3$  ; range  $y \in \mathbb{R}_e, y \neq -4$  ;

x intercept  $(1, 0)$ ; y intercept  $(0, -1)$

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$$4. \quad f(x) = \frac{x^2 + 6x + 4}{x + 3} \quad x \neq -3$$

$m > n$  denominator does not divide evenly;

Divide and forget the remainder. Set the answer equal to  $y$  to get the equation of the slanted asymptote (forget the remainder) so  $y = x + 3$  is the equation of the slanted asymptote;  $x = -3$  is the vertical asymptote; domain  $x \in \mathbb{R}_e, x \neq -3$ ; range  $y \in \mathbb{R}_e$ ;

To find the  $x$  intercept let  $y = 0$ , you will have to use the quadratic formula to find  $x$ .

$(-3 \pm \sqrt{5}, 0)$  are the  $x$  intercepts;

$y$  intercept is at  $(0, \frac{4}{3})$

$$5. \quad f(x) = \frac{2}{x^2 - 4x + 3}$$
$$f(x) = \frac{2}{(x-3)(x-1)}$$

vertical asymptotes at  $x = 3$  and  $x = 1$ ;

horizontal asymptotes at  $y = 0$  because

$m < n$ ; there is no  $x$  intercept;

$y$  intercept at  $(0, \frac{2}{3})$

$$6. \quad F(x) = \frac{(x^2 - 1)}{(x + 3)} \quad m > n \text{ the denominator}$$

does not divide evenly so there is a slanted asymptote

at  $y = x - 3$  and a vertical asymptote where  $x$  is undefined at  $x = -3$ ;  $x$  intercepts are at

$(1, 0)$  &  $(-1, 0)$ ;  $y$  intercept is at  $(0, \frac{-1}{3})$ ;

domain is  $x \in \mathbb{R}_e, x \neq -3$

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7. 
$$f(x) = \frac{(x-3)(x+1)}{(x+2)(2x-5)}$$
vertical asymptotes at  $x = -2$ ,  $x = \frac{5}{2}$  ;  
horizontal asymptotes at  $y = \frac{1}{2}$  because  
 $m = n$  ; domain  $x \in \mathbb{R}_e, x \neq -2, x \neq \frac{5}{2}$  ;  
range  $y \in \mathbb{R}_e$  ;  
x intercepts  $(3, 0)$   $(-1, 0)$  ;  
y intercept  $(0, \frac{3}{10})$

8. 
$$f(x) = \frac{(x+3)(x^2-4)}{(x+3)} \quad m > n$$
and the denominator divides evenly so there is a hole in the graph where  $x$  is undefined.

$$f(x) = \frac{(x+3)(x^2-4)}{(x+3)}$$

$g(x) = x^2 - 4$  Your graph will be a parabola with a hole in it at  $x = -3$ .  
 $g(-3) = (-3)^2 - 4 = 9 - 4 = 5$   
hole is at  $(-3, 5)$  ;  
domain is  $x \in \mathbb{R}_e, x \neq -3$  ;  
range is  $y \geq -4, y \neq 5$  ;  
x intercepts are at  $(2, 0)$  &  $(-2, 0)$  ;  
y intercept is at  $(0, -4)$   
Be sure to indicate the hole on your graph.

9. 
$$f(x) = \frac{x}{x^2-9}$$
 $m < n$  so  $y = 0$  is a horizontal asymptote;  
vertical asymptotes at  $x = -3$  &  $x = 3$  ;  
y intercept at the origin  $(0, 0)$  ;  
domain is  $x \in \mathbb{R}_e, x \neq -3, x \neq 3$  ;  
range is  $y \in \mathbb{R}_e$

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$$10. \quad f(x) = \frac{2}{x-5}$$

vertical asymptote at  $x = 5$

horizontal asymptote at  $y = 0$

domain  $x \in \mathbb{R}_e, x \neq 5$

range  $y \in \mathbb{R}_e, y \neq 0$

no x intercept

y intercept at  $x = \frac{-2}{5}$

$$11. \quad f(x) = \frac{3x-5}{2x+9}$$

vertical asymptote at  $x = \frac{-9}{2}$

horizontal asymptote at  $y = \frac{3}{2}$

domain  $x \in \mathbb{R}_e, x \neq \frac{-9}{2}$

range  $y \in \mathbb{R}_e, y \neq \frac{3}{2}$

$$12. \quad f(x) = \frac{x^2 - x - 12}{x-4}$$

$$\text{or} \quad y = \frac{(x-4)(x+3)}{(x-4)}$$

$$g(x) = x + 3$$

$$g(4) = 4 + 3 = 7$$

so there is a hole at  $(4, 7)$

domain  $x \in \mathbb{R}_e, x \neq 4$

range  $y \in \mathbb{R}_e, y \neq 7$  Be sure to indicate the hole on your graph.