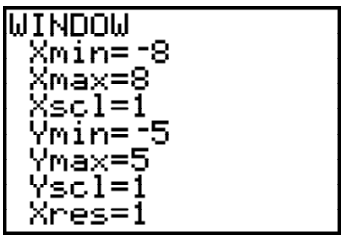
RATIONAL FUNCTIONS STUDY SHEET

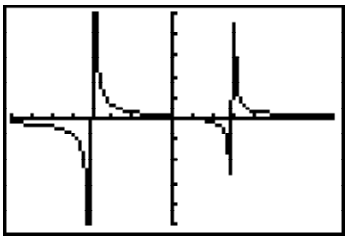
$y = f(x) = \dfrac{a_o x^m + ...}{b_o x^n + ...}$

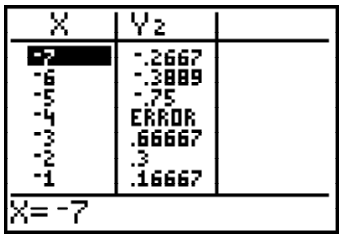
Case 1: If $m < n$ then $y = 0$ will be the horizontal asymptote, vertical asymptotes will occur where x is undefined.

example: $y = \dfrac{x - 1}{(x - 3)(x + 4)}$ horizontal asymptote: $y = 0$, vertical asymptotes: $x = 3$, $x = -4$, x intercept at $x = 1$.

Draw asymptotes, then make a T chart to draw the graph, then use a graphing calculator to verify that the graph that you drew is correct. The numerator will give you the x intercepts if you set it equal to zero. Find the x intercepts. The denominator will give you the asymptotes when set equal to zero. Find the asymptotes.





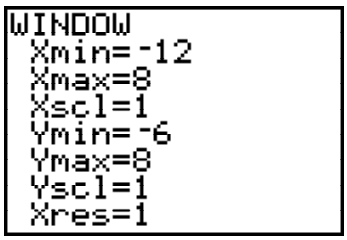


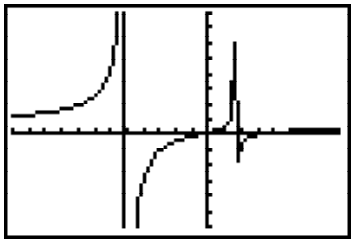
Case 2: If $m = n$ then $y = \dfrac{a_0}{bb_0}$ will be the horizontal asymptote, vertical asymptotes will occur where x is undefined.

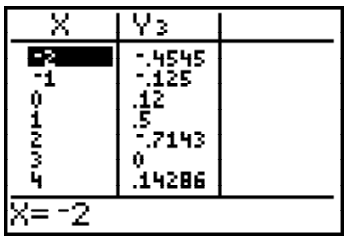
example: $y = \dfrac{(2x + 1)(x - 3)}{(x + 5)(3x - 5)}$ horizontal asymptotes: $y = \dfrac{2}{3}$, vertical asymptotes: $x = -5$, $x = \dfrac{5}{3}$

x intercepts at $x = -\frac{1}{2}$, $x = 3$.

Draw asymptotes, then make a T chart to draw the graph, then use a graphing calculator to verify that the graph that you drew is correct. The numerator will give you the x intercepts if you set it equal to zero. Find the x intercepts. The denominator will give you the asymptotes when set equal to zero.



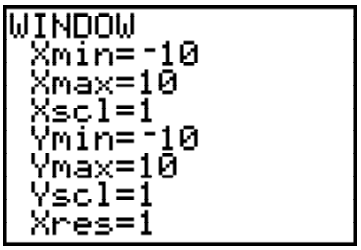


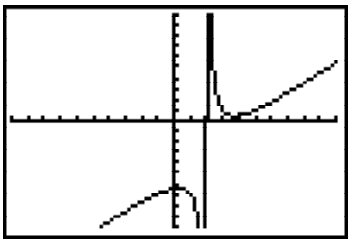


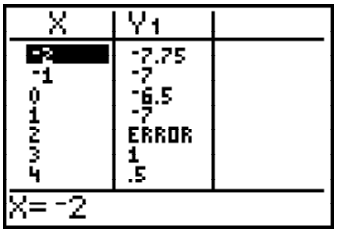
Case 3: If m > n then there will not be any horizontal asymptotes, if the denominator does not divide evenly into the numerator and m is 1 greater than n, there will be a slanted asymptote. To find the asymptote, divide the denominator into the numerator, set y equal to the answer and forget the remainder. Vertical asymptotes will occur where x is undefined.

example: $y = \frac{x^2 - 7x + 13}{x - 2}$ Slanted asymptote at $y = x - 5$, vertical asymptote at $x = 2$.

Draw asymptotes, then make a T chart to draw the graph, use a graphing calculator to verify that the graph you drew is correct. The numerator will give you the x intercepts if you set it equal to zero. You will have to use the quadratic formula if the equation does not factor.
$x = \frac{7 \pm i\sqrt{3}}{2}$ This indicated that the graph does not cross the x axis.







Case 4: if m.> n then there will be a hole in the graph if the denominator divides evenly into the numerator. The hole will occur at the x value where x is undefined. To find the y value of the hole, divide the denominator into the numerator and substitute the undefined x value in the new function and solve for the y value.

example: $y = \frac{x^2 - 7x + 10}{x - 2}$ graph will be the graph of a line with a hole in it at (2, -3)

Note when you graph the graph on the calculator you cannot see the hole in the graph. This is one of the restrictions of a graphing calculator. Check the table function of the calculator and you will note if $x = 2$ the table displays "error" when $x = 2$. When you draw the graph you must include the hole even if the calculator does not picture the hole. The hole is in the graph and you must indicate it on yours.