8.5 Finite Universe Method

An arg. is INV if it’s POSSIBLE for all the premises to be True and conclusion False. If we can find this possible case, we’ll know the arg. to be INVALID.

Back in Prop. Logic(ch.6 & 7), it was fairly easy to test using Indirect truth Tables, whether and arg. was Valid or Invalid. There was a standard method to follow.

It isn’t as easy to do with Predicate Logic.

There is a particular method that can be used called “Finite Universe Method” We start by imagining a Universe with only 1 or 2 members in it:

**One Member Univ:**
So, if we have a Univ. consisting of only 1 member (Bob), then if you take the statement “Everything is red” (x)Rx, it would be true that Bob would be red. (Rb)

Take the statement “Something is red” (∃x)Rx, it would still be true that Bob (being the only thing there) would be red. Rb

Take the statement “All dogs are happy” (x)(Dx ⊃ Hx) and apply it to our Univ. of Bob. It would then be true that If Bob is a dog, then Bob would be happy. (Db ⊃ Hb).

Take the statement “Some dogs are yappy” (∃x)(Dx · Yx) For our Universe of Bob, it would then be true that Bob is both a dog and is yappy. (Db · Yb)

**Two Member Univ:**
Now lets say we have 2 members in our Univ (Bob & Greta). Taking the same 4 statements from above, all this would be true:

**Everything is red:**  Bob is red and Greta is red (Rb · Rg)
**Something is red:** Either Bob is red OR Greta is red. (Rb v Rg)
**All dogs are happy:** If Bob is a dog, then Bob is happy AND if Greta is a dog, then Greta is happy. (Db ⊃ Hb) · (Dg ⊃ Hg)
**Some dogs are yappy:** Either Bob is a dog and yappy, OR Greta is a dog and yappy. (Db · Yb) v (Dg · Yg)

After we translate our universal and particular statements using this method, we can set it up for an Indirect truth table. The idea here is that we don’t need a universe of many things to check invalidity. If we find the possibility of invalidity in a 1 or 2 member universe, then we know for sure the arg. is Inv no matter how many other members there may be.

What you need to know for the multiple choice part of the final exam is not how to do the Indirect Truth Table, but just how to set up for it. You will be given a statement and it’s “translated form” for a Universe of two members A & B. You will need to know whether the two statements are logically equivalent, or not.
Ex. Are these two statements logically equivalent for a universe of 2 members A&B?

1. \((\exists x)(Hx \lor \neg Hx) \lor (x)(Ix \supset Mx)\) and
   \([(Ha \lor \neg Ha) \lor (Hb \lor \neg Hb)] \lor [(Ia \supset Ma) \Land (Ib \supset Mb)]\) yes

2. \((\exists x)(Ox \Land Gx) \Land (x)(Ux \lor Kx)\) and
   \([(Oa \lor Ga) \Land (Ob \lor Gb)] \Land [(Ua \lor Ka) \lor (Ub \lor Kb)]\) no (mistakes shown in red)

3. \((x)Ix \lor (\exists x)(Zx \lor Ox)\) and
   \((Ia \Land Ib) \Land [(-Za \lor Oa) \lor (Zb \lor Ob)]\) no (mistakes shown in red)

Also see the Course Packet p.36 for more on this.